Regional Mathematical Olympiad – 1992

1. Determine the set of integers n for which $n^2 + 19n + 92$ is a square.

2. If $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$, where a, b, c are positive integers with no common factor, prove that $a + b$ is a square.

3. Determine the largest 3-digit prime factor of the integer $\binom{2000}{1000}$.

4. $ABCD$ is a cyclic quadrilateral with $AC \perp BD$ and such that AC meets BD at E. Prove that
$$EA^2 + EB^2 + EC^2 + ED^2 = 4R^2,$$
where R is the radius of the circumscribing circle.

5. $ABCD$ is a cyclic quadrilateral. If x, y, z are the distances of A from the lines BD, BC, CD respectively, prove that
$$\frac{BD}{x} = \frac{BC}{y} + \frac{CD}{z}.$$

6. $ABCD$ is a quadrilateral and P, Q are midpoints of CD, AB respectively. If AP, DQ meet at X and BP, CQ meet at Y, prove that
$$\text{area of } \Delta\, ADX + \text{area of } \Delta\, BCY = \text{area of quadrilateral } PXOY.$$

7. Prove that
$$1 < \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + \cdots + \frac{1}{3001} < \frac{4}{3}.$$

8. Solve the system
$$\begin{aligned}(x+y)(x+y+z) &= 18\\ (y+z)(x+y+z) &= 30\\ (z+x)(x+y+z) &= 2A\end{aligned}$$
in terms of the parameter A.

9. The cyclic octagon $ABCDEFGH$ has sides a, a, a, a, b, b, b, b respectively. Find the radius of the circle that circumscribe $ABCDEFGH$.