Regional Mathematical Olympiad – 1993

1. Let ABC be an acute-angled triangle and CD be the altitude through C. If $AB = 8$ and $CD = 6$, find the distance between the midpoints of AD and BC.

2. Prove that the ten's digit of any power of 3 is even. (e.g. the ten's digit of $3^6 = 729$ is 2.)

3. Suppose $A_1A_2 \dots A_{20}$ is a 20-sided regular polygon. How many non-isosceles (scalene) triangles can be formed whose vertices are among the vertices of the polygon but whose sides are not the sides of the polygon?

4. Let $ABCD$ be a rectangle with $AB = a$ and $BC = b$. Suppose r_1 is the radius of the circle passing through A and B and touching CD; and similarly r_2 is the radius of the circle passing through B and C and touching AD. Show that
$$r_1 + r_2 \geq \frac{5}{8}(a + b).$$

5. Show that $19^{93} - 13^{99}$ is a positive integer divisible by 162.

6. If a, b, c, d are four positive real numbers such that $abcd = 1$, prove that $(1 + a)(1 + b)(1 + c)(1 + d) \geq 16$.

7. In a group of ten persons, each person is asked to write the sum of the ages of all the other nine persons. If all the ten sums form the 9-element set $\{82, 83, 85, 85, 87, 89, 90, 91, 92\}$, find the individual ages of the persons assuming them to be whole numbers (of years).

8. I have six friends and during a certain vacation I met them during several dinners. I found that I dined with all the six exactly on one day; with every five of them on 2 days; with every four of them on 3 days; with every three of them on 4 days; with every two of them on 5 days. Further, every friend was present at 7 dinners and every friend was absent at 7 dinners. How many dinners did I have alone?