Regional Mathematical Olympiad – 1996

1. The sides of a triangle are three consecutive integers and its inradius is 4 units. Determine the circumradius.

2. Find all triples (a, b, c) of positive integers such that
$$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)=3.$$

3. Solve for real numbers x and y:
$$\begin{aligned} xy^2 &= 15x^2+17xy+15y^2 \\ x^2y &= 20x^2+3y^2 \end{aligned}$$

4. Suppose N is an n-digit positive integer such that:
   (a) all the digits are distinct; and
   (b) the sum of any three consecutive digits is divisible by 5.

   Prove that n is at most 6. Further, show that starting with any digit one can find a 6-digit number with these properties.

5. Let ABC be a triangle and h_a be the altitude through A. Prove that
$$(b+c)^2 \geq a^2+4h_a^2.$$
   (As usual, a, b, c denote the sides BC, CA, AB respectively.)

6. Given any positive integer n, show that there are two positive rational numbers a and b, $a \neq b$, which are not integers and which are such that $a-b, a^2-b^2, a^3-b^3, \ldots, a^n-b^n$ are all integers.

7. If A is a 50-element subset of the set $\{1, 2, 3, \ldots, 100\}$ such that no two numbers from A add up to 100, show that A contains a square.