Regional Mathematical Olympiad – 1999

1. Prove that the inradius of a right-angled triangle with integer sides is an integer.

2. Find the number of positive integers which divide 10^{999} but not 10^{998}.

3. Let $ABCD$ be a square and M, N be points on sides AB, BC, respectively, such that $\angle MDN = 45°$. If R is the midpoint of MN, show that $RP = RQ$, where P, Q are the points of intersection of AC with the lines MD, ND.

4. If p, q, r are the roots of the cubic equation $x^3 - 3px^2 + 3q^2x - r^3 = 0$, show that $p = q = r$.

5. If a, b, c are the sides of a triangle, prove the following inequality:

$$\frac{a}{c+a-b} + \frac{b}{a+b-c} + \frac{c}{b+c-a} \geq 3.$$

6. Find all solutions in integers m, n of the equation

$$(m-n)^2 = \frac{4mn}{m+n-1}.$$

7. Find the number of quadratic polynomials $ax^2 + bx + c$ which satisfy the following conditions:

   (a) a, b, c are distinct;

   (b) $a, b, c \in \{1, 2, 3, \ldots, 1999\}$; and

   (c) $x + 1$ divides $ax^2 + bx + c$.