# Regional Mathematical Olympiad – 2008

**Time: 3 hours** **9 November 2008**

1. Let $ABC$ be an acute-angled triangle; and let $D, F$ be the mid-points of $BC, AB$ respectively. Let the perpendicular from $F$ to $AC$ and the perpendicular $B$ to $BC$ meet in $N$. Prove that $ND$ is equal to the circum-radius of $ABC$.

2. Prove that there exist two infinite sequences $< a_n >_{n \geq 1}$ and $< b_n >_{n \geq 1}$ of positive integers such that the following conditions hold simultaneously:

   (i) $1 < a_1 < a_2 < a_3 < \cdots;$
   (ii) $a_n < b_n < a_n^2$ for all $n \geq 1$;
   (iii) $a_n - 1$ divides $b_n - 1$ for all $n \geq 1$;
   (iv) $a_n^2 - 1$ divides $b_n^2 - 1$ for all $n \geq 1$.

3. Suppose $a$ and $b$ are real numbers such that the roots of the cubic equation $ax^3 - x^2 + bx - 1 = 0$ are all positive real numbers. Prove that

$$(i)\ 0 < 3ab \leq 1 \quad \text{and} \quad (ii)\ b \geq \sqrt{3}.$$

4. Find the number of all 6-digit natural numbers such that the sum of their digits is 10 and each of the digits $0, 1, 2, 3$ occurs at least once in them.

5. Three nonzero real numbers $a, b, c$ are said to be in harmonic progression if $\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$. Find all three-term harmonic progressions $a, b, c$ of strictly increasing positive integers in which $a = 20$ and $b$ divides $c$.

6. Find the number of all integer-sided isosceles obtuse-angled triangles with perimeter 2008.